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Theoretical study of complete contact indentations of viscoelastic materials

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Nanoindentation experiments have been used to investigate mechanical properties of polymeric thin films. In this paper, the problem is modeled as normal indentation of a viscoelastic half-space by a rigid smooth frictionless axisymmetric polynomial indenter. An analytical solution, which relates the indentation load to the penetration depth, is presented. The solution is valid as long as the contact region is simply connected and the indenter is in complete contact with the half-space. It shows that a similar fundamental relation exists in Laplace space. For a viscoelastic material with a fixed Poisson's ratio, constant-load indentation tests are direct measurements of the material's creep function, and constant-displacement tests are direct measurements of its stress relaxation function. The theory shows that the nanoindentation technique is also a useful tool to obtain viscoelastic properties of thin films.

When an axisymmetric punch indents normally into a half-space, there are two possibilities: one is that the whole punch surface contacts with the half-space; the other is that only part of the punch contacts with the half-space. Following the terminology by Gladwell [1], the first contact is called complete, and the second one is termed incomplete. In the second case, the contact pressure will drop to zero at the boundary of the contact region. Incomplete contact of a viscoelastic half-space has been treated by Lee and Radok [2], Graham [3, 4] and Ting [5, 6]. However, their formulations are generally complicated and difficult to use in the interpretation of indentation data. There are also some restrictions, e.g., the contact area does not decrease under Graham's approach. In this paper, only complete contact is considered. The main advantage of complete contacts is that the contact area is constant. This simplifies both the theoretical analysis and the experimental procedure. As the final result shows, the obtained solution is simple and easy to use for the indentation application.

We consider a rigid frictionless axisymmetric indenter with a polynomial profile and the axis of revolution as the z-axis, indenting normally into the plane $z = 0$ of a viscoelastic half-space $z \geq 0$. The problem is considered in the linear theory of viscoelasticity and the half-space is assumed to be isotropic and homogeneous.

The following equations give the relevant displacements and stresses. The vertical component of the displacement is denoted by u_z, and the stress components have two subscripts corresponding to the appropriate coordinates. $E(t)$, $J(t)$ and $\nu(t)$ are the stress relaxation function, creep function and Poisson's ratio of the viscoelastic half-space.

The governing equations for the viscoelastic half-space are:

$$2\varepsilon_{ij}(r,z,t) = u_{i,j}(r,z,t) + u_{j,i}(r,z,t) \quad \text{(Compatibility equation)} \tag{1}$$

$$\sigma_{ij}(r,z,t) = \int_{-\infty}^{t}\left[2G(t-\tau)\frac{\partial\varepsilon_{ij}(r,z,\tau)}{\partial\tau} + \delta_{ij}\lambda(t-\tau)\frac{\partial\varepsilon_{kk}(r,z,\tau)}{\partial\tau}\right]d\tau$$

(Boltzmann superposition principle), where $G(t)$ and $\lambda(t)$ are the relaxation moduli. (2)

$$\sigma_{ij,j}(r,z,t) = 0 \quad \text{(Force equilibrium)} \tag{3}$$

As Fig. 1 shows, the boundary conditions for the indentation problem are

$$\tau_{zr}(r,0,t) = \tau_{z\theta}(r,0,t) = 0, \quad (0 \leq r < \infty) \tag{4}$$

$$\sigma_{zz}(r,0,t) = 0, \quad (r > a) \tag{5}$$

$$u_z(r,0,t) = h(t) + \sum_{\alpha=\alpha_1}^{\alpha_n} a_\alpha r^\alpha, \quad (0 \leq r \leq a) \tag{6}$$

where α is a positive real number, and $h(t)$ is the indentation depth. The second term at the right hand side of Equation 6 describes the indenter shape.

Initially, no interaction happens between the indenter and the half-space, and we have the initial conditions:

$$\varepsilon_{ij}(r,z,t) = \sigma_{ij}(r,z,t) = 0, \quad (-\infty < t \leq 0) \tag{7}$$

$$h(t) = 0, \quad (-\infty < t \leq 0) \tag{8}$$

Conditions of (7) and (8) and complete contact restriction are important in the following derivation; otherwise, elastic-viscoelastic corresponding principle cannot be used directly.

Assuming Laplace transforms of all the time variables exist, we have the corresponding equations in Laplace space:

$$2\bar{\varepsilon}_{ij} = \bar{u}_{i,j} + \bar{u}_{j,i} \tag{9}$$

$$\bar{\sigma}_{ij} = 2s\bar{G}\bar{\varepsilon}_{ij} + \delta_{ij}s\bar{\lambda}\bar{\varepsilon}_{kk} \tag{10}$$

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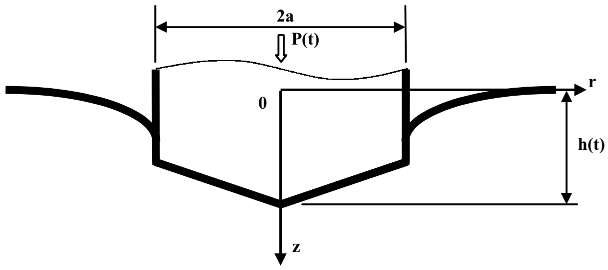

Figure 1 Normal indentation of a viscoelastic half-space.

$$\bar{\sigma}_{\mathrm{ij,j}} = 0 \tag{11}$$

$$\bar{\tau}_{\mathrm{zr}}(r, 0, s) = \bar{\tau}_{\mathrm{z}\theta}(r, 0, s) = 0, \quad (0 \le r < \infty) \tag{12}$$

$$\bar{\sigma}_{\mathrm{zz}}(r, 0, s) = 0, \quad (r > a) \tag{13}$$

$$\bar{u}_{\mathrm{z}}(r, 0, s) = \bar{h}(s) + \frac{1}{s}\sum_{\alpha=\alpha_1}^{\alpha_{\mathrm{n}}} a_\alpha r^\alpha, \quad (0 \le r \le a) \tag{14}$$

where a bar over a variable designates its Laplace transformed form, and s is the transform variable.

Comparing with its elastic counterpart [7] and using the elastic–viscoelastic corresponding principle [8], we have the vertical load in the Laplace space as

$$\bar{P} = 2a\frac{s\bar{E}\bar{h}(s)}{1-(s\bar{\nu})^2} + \sqrt{\pi}\left[\sum_{\alpha=\alpha_1}^{\alpha_{\mathrm{n}}} a_\alpha \cdot \frac{\Gamma\left(\frac{2+\alpha}{2}\right)}{\Gamma\left(\frac{3+\alpha}{2}\right)} a^{1+\alpha}\right] \times \frac{\bar{E}}{1-(s\bar{\nu})^2} \tag{15}$$

where $P(t)$ is the total load on the indenter. The time-dependent pressure distribution at the indenter—half-space interface is given in [9].

Thus, in the Laplace space, we have the following formation, which is similar to its elastic counterpart [10]:

$$\frac{\mathrm{d}\bar{P}}{\mathrm{d}\bar{h}} = 2a\frac{s\bar{E}}{1-(s\bar{\nu})^2} \tag{16}$$

If Poisson's ratio is assumed to be a constant, we have at constant load

$$h(t) = \frac{1-\nu^2}{2a}PJ(t) - \frac{\sqrt{\pi}}{2a}\left[\sum_{\alpha=\alpha_1}^{\alpha_n} a_\alpha \cdot \frac{\Gamma\left(\frac{2+\alpha}{2}\right)}{\Gamma\left(\frac{3+\alpha}{2}\right)} a^{1+\alpha}\right] \times H(t) \tag{17}$$

where $H(t)$ is the Heaviside unit step function. In the derivation, the identity $\bar{J} = \frac{1}{s^2\bar{E}}$ is used.

And, at constant displacement, we have

$$P(t) = \left\{\frac{2ah}{1-\nu^2} + \frac{\sqrt{\pi}}{1-\nu^2}\left[\sum_{\alpha=\alpha_1}^{\alpha_{\mathrm{n}}} a_\alpha \cdot \frac{\Gamma\left(\frac{2+\alpha}{2}\right)}{\Gamma\left(\frac{3+\alpha}{2}\right)} a^{1+\alpha}\right]\right\} \times E(t) \tag{18}$$

Equation 17 shows that a constant load indentation test is a direct measurement of the material's creep function; and Equation 18 shows that a constant displacement test is a direct measurement of its stress relaxation function.

The theoretical solution provides a new approach of measuring viscoelastic properties of thin films, i.e., the direct measurements of the stress relaxation function and creep function through the complete contact indentation tests.

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